Low Reynolds Number Airfoils
MAE 5233

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1 Introduction

Low Reynolds number flows concern flows with a small ratio of viscous to inertial forces. Laminar flow dominates this flow region. Low Reynolds number flight is the most common (birds, insects...) yet it proves difficult and inefficient in human controlled flight. This paper discusses and shows some characteristics of low Reynolds number flows.

2 Analysis

Flows with a chord $Re < 1,000,000$ are typically considered low Reynolds number flows. A general low Reynolds number region map is shown in Figure 1.

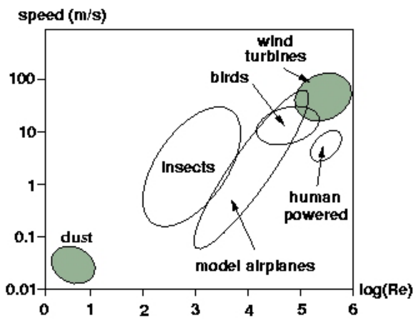

Figure 1: Low Re Region(from Filippone)[1]

2.1 Separation, Reattachment and Transition

2.1.1 von Kármán

The overall result of a bubble separation-reattachment sequence is lost momentum (increased drag). From the integral momentum von Kármán equation, the momentum loss for a boundary layer velocity jump is[2]

$$\Delta\left(\rho u_e^2 \theta\right) = -\rho u_e \delta^* \Delta u_e$$

Clearly, big bubbles (large Δu_e) cause larger drag losses than small bubbles. Small bubbles are welcome near a desired laminar-turbulent transition point, but normally we are not so lucky. Plus, an early turbulent transition could cause an increase in overall viscous drag.

2.1.2 Stratford

The Stratford relation[3] predicts laminar separation. The laminar boundary layer separation estimate is

$$(x - x_b)^2 C_p \left(\frac{dC_p}{dx}\right)^2 \approx 0.0104$$

where

$$C_p = 1 - \frac{U^2}{U_{max}^2}$$

Clearly, a typical laminar separation occurs after the maximum velocity in an unfavorable pressure gradient ($\frac{dP}{dx} > 0$).

3 Flow Interpretation and Visualization

This section attempts to show some common low Re phenomenon and design criteria.

3.1 Laminar Separation (Bubbles)

Bubbles occur when the laminar boundary layer separates from the body and reattaches downstream. Low Re flows tend to separate before transition. Figure 2 shows a schematic view of an ideal separation bubble.

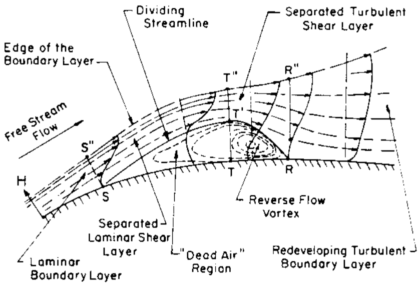

Figure 2: Laminar Separation Bubble Schematic (from Roberts)[4]

As discussed above, the bubble tends to create a turbulent transition and thus a velocity jump. Figure 3 shows the momentum and velocity distributions across a bubble.

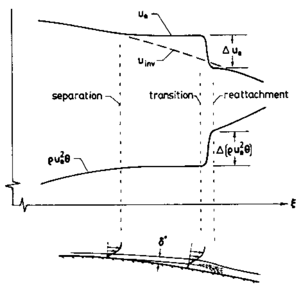

Figure 3: Laminar Separation Profile(from Drela)[2]

An actual photo of a separation bubble is given in Figure 4. Notice the smooth "dead air" region at the center and the turbulent transition at the far right.

3.2 Design

The low Re airfoil design is complicated. No general closed-form analytical solution exists [1]. Ideally, a (perfect) Navier-Stokes analysis would be used for

Figure 4: Laminar Separation Bubble (from Cole)[5]

design; however, N-S analysis is usually not practical.

Geometric and aerodynamic effects of a low Re airfoil (at least for the initial design) are predictable using some rules of thumb and a sense of low Re physics. Figures 5 and 6 and Table 1 give some typical tradeoffs and design limitations. Obviously, there is no single *best* airfoil.

Increasing	Increases	Decreases
Ramp length	C_m	Bubble loss, friction drag $C_{L_{max}}$
Ramp slope	Poor surface degradation	C_M, bubble loss, $C_{L_{max}}$
Ramp arch	Bubble loss, $C_{L_{max}}$ α range	Poor surface degradation
Bottom loading	C_M, $C_{L_{max}}$	Thickness, α range
Recovery concavity	$C_{L_{max}}$, bubble loss	Aft thickness, drag creep
Thickness	Drag, α range	Structural weight
Leading edge radius	Drag imperfection tolerance	α range
Trailing edge angle	Manufactoring ease	$C_{L_{max}}$

Table 1: Design Parameters (from Drela)[2]

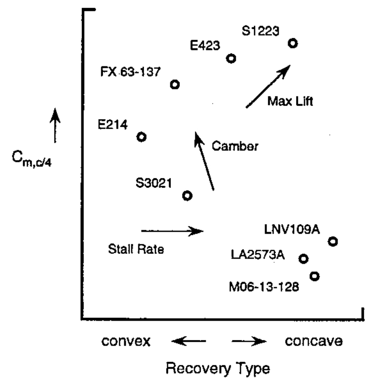

Figure 5: Design Trends (from Selig)[6]

[1]If a general solution existed, it would probably be unreasonably complex!

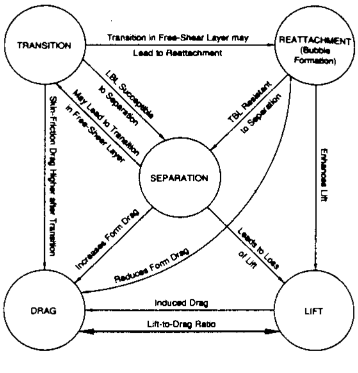

Figure 6: Design Relationships (from Cole)[5]

The final-design airfoil geometry obviously depends on the application; however, a typical shape is that of the Eppler 423 shown in Figure 7.

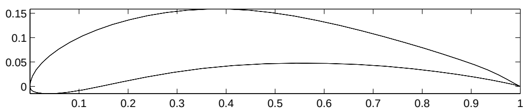

Figure 7: Eppler 423

Figure 8 shows the drag polar for the Eppler 423 airfoil for 3 values of Re. Notice how the separation

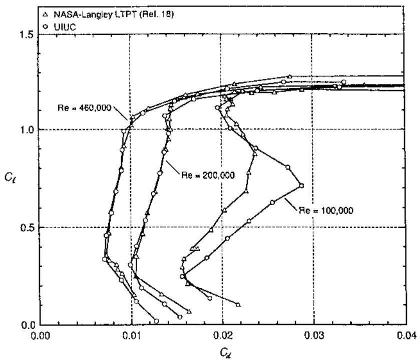

Figure 8: Eppler 387 Drag Polar (from Selig)[7]

bubbles increase the drag at zero angle of attack for the Re = 100000 flow. In fact, one-off airfoils designed for a particular mission may not perform adequately when off-design. Transonic low Re airfoils are particularly sensitive to off-design operation. A shock-BL interaction dramatically increases the BL thickness in Figure 9.

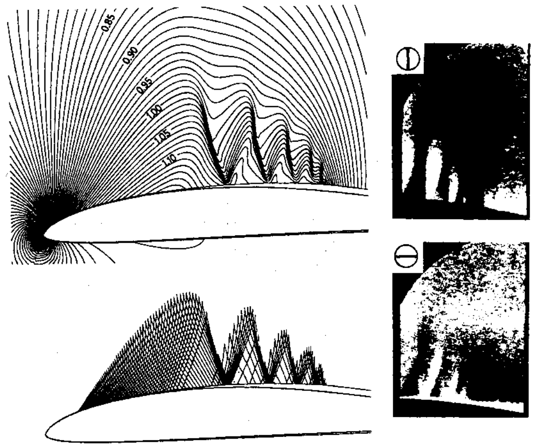

Figure 9: Mach contours for a transonic (M=0.65) Eppler 387 at Re = 200000 (from Drela)[8]

The Clark Y airfoil, Figure 10, was not designed for low Re. The figure shows the typical C_P distribution until approximately a length of 0.7 chords back from the leading edge. At this location, the C_P levels off and later suddenly drops off. This leveling-off is characteristic of a laminar separation bubble. Due to the laminar bubble, the flow outside the boundary layer *feels* a thicker airfoil and adjusts its velocity distribution accordingly. Of course, the bubble acts as a turbulent trip.

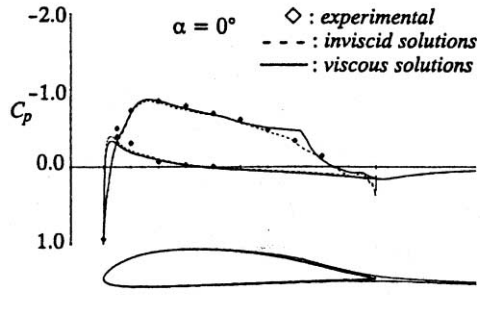

Figure 10: Clark Y C_P at $\alpha = 0$ (from Shyy)[9]

An overwhelming number of papers address low Re flows. The reference section below is a good start for further information. Also, major contributors such as Selig, Wortmann and Eppler are discussed in many technical papers. An excellent resource for specific airfoil design and mental calibration is the XFOIL computer program.

References

[1] A. Filippone, "http://aerodyn.org/lowspeed /lowspeed.html," Nov 2001.

[2] M. Drela, "Low-reynolds-number airfoil design for the M I T daedalus prototype: A case study," *J. of Aircraft*, vol. 25, pp. 724–732, 1988.

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[5] G. Cole and T. Mueller, "http:// amber.aae.uiuc.edu/ m-selig/gifs/ndbub.jpg," Nov 2001.

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[9] e. a. Wei Shyy, "Rigid and flexible low reynolds number airfoils," *J. of Aircraft*, vol. 36, pp. 523–529, May-June 1999.