1 Introduction

Low Reynolds number flows concern flows with a small ratio of viscous to inertial forces. Laminar flow dominates this flow region. Low Reynolds number flight is the most common (birds, insects...) yet it proves difficult and inefficient in human controlled flight. This paper discusses and shows some characteristics of low Reynolds number flows.

2 Analysis

Flows with a chord $Re < 1,000,000$ are typically considered low Reynolds number flows. A general low Reynolds number region map is shown in Figure 1.

![Figure 1: Low $Re$ Region(from Filippone)[1]](image)

2.1 Separation, Reattachment and Transition

2.1.1 von Kármán

The overall result of a bubble separation-reattachment sequence is lost momentum (increased drag). From the integral momentum von Kármán equation, the momentum loss for a boundary layer velocity jump is

$$\Delta \left( \rho u^2 \theta \right) = -\rho u e \delta^* \Delta u_e$$

Clearly, big bubbles (large $\Delta u_e$) cause larger drag losses than small bubbles. Small bubbles are welcome near a desired laminar-turbulent transition point, but normally we are not so lucky. Plus, an early turbulent transition could cause an increase in overall viscous drag.

2.1.2 Stratford

The Stratford relation[3] predicts laminar separation. The laminar boundary layer separation estimate is

$$(x - x_b)^2 C_p \left( \frac{dC_p}{dx} \right)^2 \approx 0.0104$$

where

$$C_p = 1 - \frac{U^2}{U_{max}^2}$$

Clearly, a typical laminar separation occurs after the maximum velocity in an unfavorable pressure gradient $(\frac{dP}{dx} > 0)$.

3 Flow Interpretation and Visualization

This section attempts to show some common low $Re$ phenomenon and design criteria.
3.1 Laminar Separation (Bubbles)

Bubbles occur when the laminar boundary layer separates from the body and reattaches downstream. Low Re flows tend to separate before transition. Figure 2 shows a schematic view of an ideal separation bubble.

As discussed above, the bubble tends to create a turbulent transition and thus a velocity jump. Figure 3 shows the momentum and velocity distributions across a bubble.

An actual photo of a separation bubble is given in Figure 4. Notice the smooth “dead air” region at the center and the turbulent transition at the far right.

3.2 Design

The low Re airfoil design is complicated. No general closed-form analytical solution exists. Ideally, a (perfect) Navier-Stokes analysis would be used for design; however, N-S analysis is usually not practical.

Geometric and aerodynamic effects of a low Re airfoil (at least for the initial design) are predictable using some rules of thumb and a sense of low Re physics. Figures 5 and 6 and Table 1 give some typical tradeoffs and design limitations. Obviously, there is no single best airfoil.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Increasing</th>
<th>Decreases</th>
</tr>
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<tbody>
<tr>
<td>Ramp length</td>
<td>$C_m$</td>
<td>Bubble loss, friction drag $C_{L_{max}}$</td>
</tr>
<tr>
<td>Ramp slope</td>
<td>Poor surface degradation</td>
<td>$C_M$, bubble loss, $C_{L_{max}}$</td>
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<tr>
<td>Ramp arch</td>
<td>Bubble loss, $C_{L_{max}}$</td>
<td>Poor surface degradation</td>
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<tr>
<td>Bottom loading</td>
<td>$C_M$, $C_{L_{max}}$</td>
<td>Thickness, $\alpha$ range</td>
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<tr>
<td>Recovery concavity</td>
<td>$C_{L_{max}}$, bubble loss</td>
<td>Aft thickness, drag creep</td>
</tr>
<tr>
<td>Thickness</td>
<td>Drag, $\alpha$ range</td>
<td>Structural weight</td>
</tr>
<tr>
<td>Leading edge radius</td>
<td>Drag imperfection tolerance</td>
<td>$\alpha$ range</td>
</tr>
<tr>
<td>Trailing edge angle</td>
<td>Manufacturing ease</td>
<td>$C_{L_{max}}$</td>
</tr>
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Table 1: Design Parameters (from Drela)
The final-design airfoil geometry obviously depends on the application; however, a typical shape is that of the Eppler 423 shown in Figure 7.

Figure 7: Eppler 423

Figure 8 shows the drag polar for the Eppler 423 airfoil for 3 values of Re. Notice how the separation bubbles increase the drag at zero angle of attack for the Re = 100000 flow. In fact, one-off airfoils designed for a particular mission may not perform adequately when off-design. Transonic low Re airfoils are particularly sensitive to off-design operation. A shock-BL interaction dramatically increases the BL thickness in Figure 9.

Figure 8: Eppler 387 Drag Polar (from Selig)[7]

The Clark Y airfoil, Figure 10, was not designed for low Re. The figure shows the typical Cp distribution until approximately a length of 0.7 chords back from the leading edge. At this location, the Cp levels off and later suddenly drops off. This leveling-off is characteristic of a laminar separation bubble. Due to the laminar bubble, the flow outside the boundary layer feels a thicker airfoil and adjusts its velocity distribution accordingly. Of course, the bubble acts as a turbulent trip.

Figure 10: Clark Y Cp at α = 0 (from Shyy)[9]

An overwhelming number of papers address low Re flows. The reference section below is a good start for further information. Also, major contributors such as Selig, Wortmann and Eppler are discussed in many technical papers. An excellent resource for specific airfoil design and mental calibration is the XFOIL computer program.
References


