CFD-BASED AEROSERVOELASTIC PREDICTIONS ON A BENCHMARK
CONFIGURATION USING THE TRANSPIRATION METHOD

ERRATA

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Abstract: This document outlines all known technical errors in the Masters Thesis entitled CFD-Based Aeroservoelastic Predictions on a Benchmark Configuration Using the Transpiration Method by Cole H. Stephens.

Known Errors:

- *Pages 62-64:* *[TJC]*

The derivation outlined on pages 62–63 is correct for a scalar equation representing a generalized moment. However, this scalar transformation cannot be applied directly to the complete vector equation of motion as presented on pages 63–64. The vector equation of motion requires a derivation with a matrix transformation in order to correctly transform the off-diagonal coupling terms. First, we define a matrix transformation that converts the vector of three generalized degrees of freedom, $\{q_1, q_2, q_3\}^T$, to the vector of three structural degrees of freedom, $\{h, \alpha, \delta\}^T$, representing plunge, pitch and flap deflection respectively. This matrix transformation is defined as follows:

$$\begin{Bmatrix} h \\ \alpha \\ \delta \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\pi}{180} & 0 \\ 0 & 0 & \frac{\pi}{180} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}$$

Next, we define a matrix transformation that converts the vector of three generalized forces, $\{G_1, G_2, G_3\}^T$, to the vector of three aerodynamic forces, $\{f_1, f_2, f_3\}^T$, representing lift, pitch moment and hinge moment respectively. This matrix transformation is defined as follows:

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{180}{\pi} & 0 \\ 0 & 0 & \frac{180}{\pi} \end{bmatrix} \begin{Bmatrix} G_1 \\ G_2 \\ G_3 \end{Bmatrix}$$

Finally, the above relationships are substituted into the vector equation of motion for the system to derive a consistent definition for the generalized equation of motion. The basic equation of motion for the system is defined as follows:

$$\begin{bmatrix} m & S_\alpha & S_{\delta,h} \\ S_\alpha & I_\alpha & S_{\delta,\alpha} \\ S_{\delta,h} & S_{\delta,\alpha} & I_\delta \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\delta} \end{Bmatrix} + \begin{bmatrix} g_h & 0 & 0 \\ 0 & g_\alpha & 0 \\ 0 & 0 & g_\delta \end{bmatrix} \begin{Bmatrix} \dot{h} \\ \dot{\alpha} \\ \dot{\delta} \end{Bmatrix} + \begin{bmatrix} k_h & 0 & 0 \\ 0 & k_\alpha & 0 \\ 0 & 0 & k_\delta \end{bmatrix} \begin{Bmatrix} h \\ \alpha \\ \delta \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix}$$

Substituting the matrix transformation for the generalized degrees of freedom produces the following expression:

$$\begin{bmatrix} m & S_\alpha & S_{\delta,h} \\ S_\alpha & I_\alpha & S_{\delta,\alpha} \\ S_{\delta,h} & S_{\delta,\alpha} & I_\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\pi}{180} & 0 \\ 0 & 0 & \frac{\pi}{180} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} g_h & 0 & 0 \\ 0 & g_\alpha & 0 \\ 0 & 0 & g_\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\pi}{180} & 0 \\ 0 & 0 & \frac{\pi}{180} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix} + \dots$$

$$\dots + \begin{bmatrix} k_h & 0 & 0 \\ 0 & k_\alpha & 0 \\ 0 & 0 & k_\delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\pi}{180} & 0 \\ 0 & 0 & \frac{\pi}{180} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{180}{\pi} & 0 \\ 0 & 0 & \frac{180}{\pi} \end{bmatrix} \begin{Bmatrix} G_1 \\ G_2 \\ G_3 \end{Bmatrix}$$

Combining the constant coefficient and transformation matrices in the above expression yields the following definition for the generalized mass, damping and stiffness:

$$\begin{bmatrix} m & \frac{\pi}{180}S_\alpha & \frac{\pi}{180}S_{\delta,h} \\ \frac{\pi}{180}S_\alpha & \left(\frac{\pi}{180}\right)^2 I_\alpha & \left(\frac{\pi}{180}\right)^2 S_{\delta,\alpha} \\ \frac{\pi}{180}S_{\delta,h} & \left(\frac{\pi}{180}\right)^2 S_{\delta,\alpha} & \left(\frac{\pi}{180}\right)^2 I_\delta \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} g_h & 0 & 0 \\ 0 & \left(\frac{\pi}{180}\right)^2 g_\alpha & 0 \\ 0 & 0 & \left(\frac{\pi}{180}\right)^2 g_\delta \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix} + \ldots$$

$$\ldots + \begin{bmatrix} k_h & 0 & 0 \\ 0 & \left(\frac{\pi}{180}\right)^2 k_\alpha & 0 \\ 0 & 0 & \left(\frac{\pi}{180}\right)^2 k_\delta \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} G_1 \\ G_2 \\ G_3 \end{Bmatrix}$$

Notice that the only change between the original derivation and the one presented here is in the coupling terms of the generalized mass matrix for the plunge degree of freedom. The new mass matrix is now symmetric across the primary diagonal.

- *Page 65:* *[TJC]*

  Based on the above derivation, the generalized mass matrix in Table 3-5 should read as follows:

Generalized Mass Matrix

5.06670E-01	8.84300E-04	5.02655E-05
8.84300E-04	1.02350E-02	5.73900E-06
5.02655E-05	5.73900E-06	4.01340E-02