CONVERTING GENERALIZED FORCES TO AERODYNAMIC MOMENTS

Nomenclature:

$\mathbf{M}$ = aerodynamic moment vector (contains roll, pitch, and yaw moments)
P_i = aerodynamic pressure at node i
A_i = reference area for node i
$\mathbf{n}_i$ = normal vector for node i
$\mathbf{r}_i$ = coordinate vector for node i
$\mathbf{\Phi}_i$ = modal displacement vector for node i
G = generalized force
N = number of surface nodes
θ = "small" angle of rotation (in radians) used to define a particular mode shape

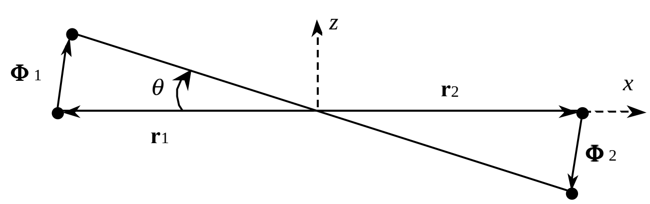
Definition of aerodynamic moment vector:

(1)
$$\mathbf{M} = \sum_{i=1}^{N} (P_i \cdot A_i \cdot \mathbf{n}_i) \times \mathbf{r}_i = \sum_{i=1}^{N} P_i \cdot A_i \cdot \begin{Bmatrix} n_z r_y - n_y r_z \\ n_x r_z - n_z r_x \\ n_y r_x - n_x r_y \end{Bmatrix}$$

Definition of STARS generalized force:

(2)
$$G = \sum_{i=1}^{N} (P_i \cdot A_i \cdot \mathbf{n}_i) \cdot \mathbf{\Phi}_i$$

Geometry Layout:



Definition of nodal displacement vector representing a pitch mode:

(3)
$$\mathbf{\Phi}_i = \begin{bmatrix} \cos\theta - 1 & 0 & \sin\theta \\ 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta - 1 \end{bmatrix} \begin{Bmatrix} r_x \\ r_y \\ r_z \end{Bmatrix}_i \approx \begin{Bmatrix} \theta \cdot r_z \\ 0 \\ -\theta \cdot r_x \end{Bmatrix}_i$$
(assuming small θ)

Relationship between aerodynamic moment and STARS generalized force:

(4)
$$G = \sum_{i=1}^{N} \left(P_i \cdot A_i \cdot \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}_i \right) \cdot \begin{Bmatrix} \theta \cdot r_z \\ 0 \\ -\theta \cdot r_x \end{Bmatrix}_i = \theta \cdot \sum_{i=1}^{N} P_i \cdot A_i \cdot (n_x r_z - n_z r_x)_i$$

(5)
$$M_y = \frac{G}{\theta}$$