Linear ARMA Aerodynamics Model

The linear ARMA model predicts aerodynamic forces based on previous forces and motions. The form consists of two summations of system-identification coefficients and the corresponding forces and motions.

$$f(k) = \sum_{i=1}^{na} [A_i] f(k-i) + \sum_{i=0}^{nb-1} [B_i] q(k-i)$$

The aerodynamics state vector is,

$$x_a(k) = \begin{bmatrix} f(k-1) \\ \vdots \\ f(k-na) \\ q(k-1) \\ \vdots \\ q(k-nb+1) \end{bmatrix}$$

The discrete-time state space form is,

$$x_a(k+1) = [G_a] x_a(k) + [H_a] q(k)$$

$$f(k) = [C_a] x_a(k) + [D_a] q(k) + f_0$$

$$[G_a] = \begin{bmatrix} [A_1] & [A_2] & \cdots & [A_{na-1}] & [A_{na}] & [B_1] & [B_2] & \cdots & [B_{nb-2}] & [B_{nb-1}] \\ [I] & [0] & \cdots & [0] & [0] & [0] & [0] & \cdots & [0] & [0] \\ [0] & [I] & \cdots & [0] & [0] & [0] & [0] & \cdots & [0] & [0] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & [0] & [0] \\ [0] & [0] & \cdots & [I] & [0] & [0] & [0] & \cdots & [0] & [0] \\ [0] & [0] & \cdots & [0] & [0] & [0] & [0] & \cdots & [0] & [0] \\ [0] & [0] & \cdots & [0] & [0] & [I] & [0] & \cdots & [0] & [0] \\ [0] & [0] & \cdots & [0] & [0] & [0] & [I] & \cdots & [0] & [0] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] & [0] & [0] & \cdots & [I] & [0] \end{bmatrix} \qquad [H_a] = \begin{bmatrix} [B_0] \\ [0] \\ [0] \\ \vdots \\ [0] \\ [I] \\ [0] \\ [0] \\ \vdots \\ [0] \end{bmatrix}$$

$$[C_a] = [[A_1] \quad [A_2] \quad \cdots \quad [A_{na-1}] \quad [A_{na}] \quad [B_1] \quad [B_2] \quad \cdots \quad [B_{nb-2}] \quad [B_{nb-1}]] \qquad [D_a] = [B_0]$$