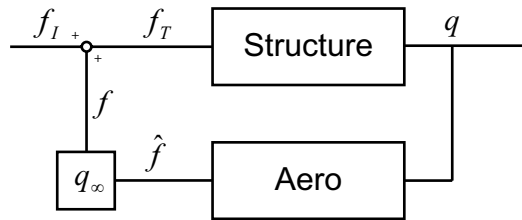


## Coupled Aeroelastic Model



### Structural Model

$$x_s(k+1) = [G_s]x_s(k) + [H_s]f_T(k)$$

$$q(k) = [C_s]x_s(k)$$

### Aerodynamics Model

$$x_a(k+1) = [G_a]x_a(k) + [H_a]q(k)$$

$$\hat{f}(k) = [C_a]x_a(k) + [D_a]q(k) + \hat{f}_0$$

### Aeroelastic Coupling

$$f_T = f + f_I$$

$$f = q_\infty \hat{f}$$

Substitute structures model into aerodynamics model

$$x_a(k+1) = [G_a]x_a(k) + [H_a][C_s]x_s(k)$$

$$\hat{f}(k) = C_a x_a(k) + D_a C_s x_s(k) + \hat{f}_0$$

Add aeroelastic coupling into structures

$$x_s(k+1) = G_s x_s(k) + H_s (q_\infty \hat{f} + f_I)$$

Add aerodynamics resultant into structures

$$x_s(k+1) = G_s x_s(k) + q_\infty H_s (C_a x_a(k) + D_a C_s x_s(k) + \hat{f}_0 + f_I)$$

Reduce and collect terms

$$x_s(k+1) = (G_s + q_\infty H_s D_a C_s)x_s(k) + q_\infty H_s C_a x_a(k) + q_\infty H_s \hat{f}_0 + H_s f_I$$

### Aeroelastic State Space Form

$$\begin{bmatrix} x_s(k+1) \\ x_a(k+1) \end{bmatrix} = \begin{bmatrix} G_s + q_\infty H_s D_a C_s & q_\infty H_s C_a \\ H_a C_s & G_a \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_a(k) \end{bmatrix} + \begin{bmatrix} H_s \\ 0 \end{bmatrix} f_I + \begin{bmatrix} q_\infty H_s \hat{f}_0 \\ 0 \end{bmatrix}$$

$$q(k) = \begin{bmatrix} [C_s] & [0] \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_a(k) \end{bmatrix}$$