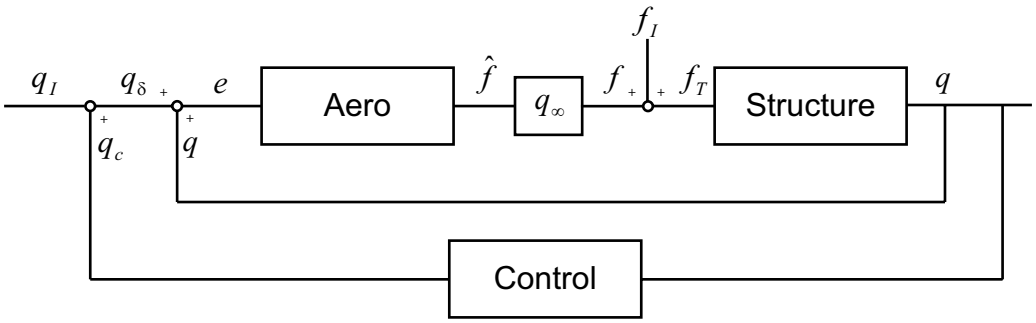
Aeroservoelastic Model



Structure

$$x_s(k+1) = [G_s]x_s(k) + [H_s]f_T(k)$$
$$q(k) = [C_s]x_s(k)$$

Aerodynamics

$$x_a(k+1) = [G_a]x_a(k) + [H_a]q(k)$$
$$\hat{f}(k) = [C_a]x_a(k) + [D_a]q(k) + \hat{f}_0$$

Control Algorithm

$$q_c(k) = [K]q(k)$$

Aeroservoelastic Coupling

$$f_T = f + f_I$$
$$f = q_\infty \hat{f}$$
$$q_\delta = q_I + q_c$$
$$e = q_\delta + q$$

Add structures into aerodynamics

$$x_a(k+1) = G_a\, x_a(k) + H_a\, e(k)$$
$$\hat{f}(k) = C_a\, x_a(k) + D_a\, e(k) + \hat{f}_0$$

Add aeroservoelastic coupling into aerodynamics resultant

$$x_a(k+1) = G_a\, x_a(k) + H_a\left(q_I(k) + q_c(k) + C_s\, x_s(k)\right)$$
$$\hat{f}(k) = C_a\, x_a(k) + D_a\left(q_I(k) + q_c(k) + C_s\, x_s(k)\right) + \hat{f}_0$$

Add control algorithm

$$x_a(k+1) = G_a\, x_a(k) + H_a\left(q_I(k) + KC_s\, x_s(k) + C_s\, x_s(k)\right)$$
$$\hat{f}(k) = C_a\, x_a(k) + D_a\left(q_I(k) + KC_s\, x_s(k) + C_s\, x_s(k)\right) + \hat{f}_0$$

Reduce and collect terms

$$x_a(k+1) = G_a\, x_a(k) + H_a\left[\mathrm{I}+K\right]C_s\, x_s(k) + H_a q_I(k)$$

$$\hat{f}(k) = C_a\, x_a(k) + D_a\left[\mathrm{I}+K\right]C_s\, x_s(k) + D_a q_I(k) + \hat{f}_0$$

Add coupling into structures

$$x_s(k+1) = G_s\, x_s(k) + H_s\left(q_\infty \hat{f}(k) + f_I(k)\right)$$

Add aero resultant into structures

$$x_s(k+1) = G_s\, x_s(k) + q_\infty H_s\left(C_a\, x_a(k) + D_a\left[\mathrm{I}+K\right]C_s\, x_s(k) + D_a q_I(k) + \hat{f}_0\right) + H_s f_I(k)$$

Reduce and collect terms

$$x_s(k+1) = \left(G_s + q_\infty H_s\, D_a\left[\mathrm{I}+K\right]C_s\right)x_s(k) + q_\infty H_s C_a\, x_a(k) + q_\infty H_s D_a q_I(k) + q_\infty H_s \hat{f}_0 + H_s f_I$$

Aeroservoelastic State Space Form

$$\begin{bmatrix} x_s(k+1) \\ x_a(k+1) \end{bmatrix} = \begin{bmatrix} G_s + q_\infty H_s\, D_a\left[\mathrm{I}+K\right]C_s & q_\infty H_s C_a \\ H_a\left[\mathrm{I}+K\right]C_s & G_a \end{bmatrix}\begin{bmatrix} x_s(k) \\ x_a(k) \end{bmatrix} + \begin{bmatrix} H_s \\ 0 \end{bmatrix} f_I + \begin{bmatrix} q_\infty H_s D_a \\ H_a \end{bmatrix} q_I + \begin{bmatrix} q_\infty H_s \\ 0 \end{bmatrix}\hat{f}_0$$

$$q(k) = \left[\left[C_s\right]\left[0\right]\right]\begin{bmatrix} x_s(k) \\ x_a(k) \end{bmatrix}$$