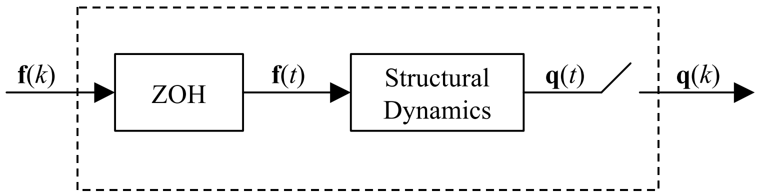
Discrete-Time Structural Model



Equivalent Discrete-Time Structure

The generalized structural equation of motion is…

$$[\mathbf{M}]\ddot{\mathbf{q}}+[\mathbf{C}]\dot{\mathbf{q}}+[\mathbf{K}]\mathbf{q}=\mathbf{f}(t)$$

We can define a state vector…

$$\mathbf{x}_s(t)=\begin{Bmatrix}\dot{\mathbf{q}}(t)\\ \mathbf{q}(t)\end{Bmatrix}$$

Then the state-space form for the continuous-time structural model is…

$$\dot{\mathbf{x}}_s(t)=[\mathbf{A}_s]\mathbf{x}_s(t)+[\mathbf{B}_s]\mathbf{f}(t)$$

$$\mathbf{q}(t)=[\mathbf{C}_s]\mathbf{x}_s(t)+[\mathbf{D}_s]\mathbf{f}(t)$$

where…

$$[\mathbf{A}_s]=\begin{bmatrix}-[\mathbf{M}]^{-1}[\mathbf{C}] & -[\mathbf{M}]^{-1}[\mathbf{K}]\\ [\mathbf{I}] & [0]\end{bmatrix} \qquad [\mathbf{B}_s]=\begin{bmatrix}[\mathbf{M}]^{-1}\\ [0]\end{bmatrix}$$

$$[\mathbf{C}_s]=[[0] \quad [\mathbf{I}]] \qquad [\mathbf{D}_s]=[0]$$

We can then convert this to the zero-order hold discrete-time equivalent…

$$\mathbf{x}_s(k+1)=[\mathbf{G}_s]\mathbf{x}_s(k)+[\mathbf{H}_s]\mathbf{f}(k)$$

$$\mathbf{q}(k)=[\mathbf{C}_s]\mathbf{x}_s(k)+[\mathbf{D}_s]\mathbf{f}(k)$$

where…

$$[\mathbf{G}_s]=e^{[\mathbf{A}_s]\Delta t} \qquad [\mathbf{H}_s]=\left(\int_0^{\Delta t} e^{[\mathbf{A}_s]\lambda}d\lambda\right)[\mathbf{B}_s]=\left(e^{[\mathbf{A}_s]\Delta t}-[\mathbf{I}]\right)[\mathbf{A}_s]^{-1}[\mathbf{B}_s]$$

and [$\mathbf{C_s}$] and [$\mathbf{D_s}$] do not change.