Abstract

Modern high performance aerospace vehicles are particularly susceptible to destructive fluid-structure interactions. Accurate and timely aerodynamic predictions are needed for efficient vehicle design and evaluation. System identification offers an efficient and powerful prediction methodology by substituting a trained mathematical system model for the actual aerodynamic system. Coupling the system model with structural and control systems allows for fast and intuitive vehicle analysis. The challenge becomes determining a system model that accurately represents the dominant fluid-flow physics. This thesis investigated linear aerodynamic system identification for aeroservoelastic predictions based on Computational Fluid Dynamics (CFD) flow predictions.
Improved System Identification for Aeroservoelastic Predictions

Presented by
Charles Robert O'Neill

School of Mechanical and Aerospace Engineering
Oklahoma State University

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Aeroservoelasticity is a combination of structural, aerodynamics and controls systems.
Structural System

The structural system properties are determined through decomposing the overall structure into modeshapes and frequencies.

**6 mode plate example**

Mode 1: 589 Hz  
Mode 2: 762 Hz  
Mode 3: 1071 Hz  
Mode 4: 1516 Hz  
Mode 5: 1533 Hz  
Mode 6: 1702 Hz

**Governing Equation**

The generic structural governing equation contains mass, damping, stiffness and an external forcing function.

\[
[M] \ddot{q} + [C] \dot{q} + [K]q = F
\]

The discrete time structural model is the following:

\[
x_s(k+1) = [G_s] x_s(k) + [H_s] F(k)
\]

\[
q(k) = [C_s] x_s(k) + [D_s] F(k)
\]
The control system properties are determined through the designer's specific system performance criteria. A control system modifies the overall system response.

Determining a useful controls model is conceptually straightforward. The exact implementation methodology can vary. A generalized control law follows:

$$\eta = [K] \tilde{x}$$
The Problem

Traditional free response analysis of aerospace vehicles with Computational Fluid Dynamics requires significant computing power and does not allow for intuitive or quick analysis and design.

Objective

The objective is to develop an improved system identification methodology for aeroservoelastic predictions. Substituting the trained aerodynamic system model for the actual aerodynamic system will allow for an efficient and powerful prediction methodology.
Decompose into 4 areas

Aerodynamic System Model
- How to represent the aerodynamic system

Training Method
- How to determine the aerodynamic system parameters from raw data

Excitation Signal
- How to excite the dominate aerodynamics

Performance Criteria
- How to evaluate and select a system model
Unsteady Aerodynamics

No general closed form solution exists for unsteady aerodynamics. Two classical unsteady aerodynamic expressions result from assuming an incompressible, inviscid flow with simple motions and simple geometry. The Wagner, Theodorsen and wave-equation expressions allow insights into representing unsteady aerodynamic

**Wagner Step Input**

**Theodorsen Harmonic Motion**

\[ L = \pi \rho b^2 (\ddot{h} + U \dot{\alpha} - ba \ddot{\alpha}) + 2\pi b \rho U (\dot{h} + U \alpha - ba \dot{\alpha})C(k) \]

**Wave Equation**

\[ \frac{D^2}{Dt^2} = a_0^2 \nabla^2 \]

- No Motion
- Subsonic
- Supersonic
Aerodynamic Modeling Requirements

- Consistent Boundary Conditions
- Input-Output Dynamics
- Starting and Ending Conditions
- Accuracy near system stability point
- Captures different time scales
- Motion limitations to remain in linear region
Aerodynamic Representations

Using the previous unsteady aerodynamic results and modeling requirements, the following generic aerodynamic function is proposed.

\[ f^{(n)} + \ldots + \dot{f} + \ddot{f} + f + f(\text{delays}) = x^{(n)} + \ldots + \dot{x} + \ddot{x} + x + x(\text{delays}) + C \]

Now, a specific system model needs to be selected. The best candidates were the indicial response model and the ARMA model.

Indicial Response

The indicial response consists of a step response. The system response is determined through convolution. The system parameters are determined by the step response function \( g(t) \). Determining \( g(t) \) becomes complicated when the boundary conditions are coupled.

\[ y(t) = \int_{0}^{t} g(t - \tau)u(t)d\tau \]

ARMA Model

The Auto Regressive Moving Average (ARMA) model consists of an input-output expression with internal and input responses. Individual coefficients \( A_i \) and \( B_i \) determine the system parameters. The ARMA model directly corresponds to a discrete-time ODE representation.

\[ f(k) = \sum_{i=1}^{na} [A_i] f(k - i) + \sum_{i=0}^{nb-1} [B_i] q(k - i) \]

Internal Response \hspace{0.5cm} Input Response
Training Methodology

The training method reduces the raw aerodynamic time histories to usable aerodynamic system models.

Expressing the inputs and outputs in the ARMA system form yields:

\[
\begin{bmatrix}
\text{Previous Outputs} \\
\vdots \\
y(t-1) \quad \cdots \quad y(t-na)
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
\vdots \\
x_i(t) \quad \cdots \\
x_n(t-nb-1) \quad \cdots \\
x_m(t-nb-1)
\end{bmatrix}
\begin{bmatrix}
\text{Past and Current Inputs} \\
\vdots
\end{bmatrix}
\begin{bmatrix}
A_i \\
B_i
\end{bmatrix}
= \begin{bmatrix}
\text{Current Output} \\
\vdots
\end{bmatrix}
\]

This is a linear equation of the form:

\[
[A]{x} = {b}
\]

Solving for the \(x\) vector yields the ARMA model coefficients. Singular value decomposition (SVD) is used to solve this linear least squares algebraic system. SVD is preferred because of its robustness and its ability to solve overdetermined systems.
Parallel Training

Parallel training is possible by making noticing the structure of the training data matrix. The columns represent the model order, the rows represent the individual timesteps.

\[
\begin{bmatrix}
y(0) & x(1) & x(2) \\
y(1) & x(2) & x(3) \\
y(2) & x(3) & x(4)
\end{bmatrix}
\cdot
\begin{bmatrix}
A_1 \\
B_1 \\
B_2
\end{bmatrix}
= 
\begin{bmatrix}
y(1) \\
y(2) \\
y(3)
\end{bmatrix}
\]

Catenating additional rows onto the matrix is permitted. The only caveat is to ensure consistency of previous forces and displacements occurring before time zero.

Advantages:
- Eliminates long-lag contamination
- Eliminates modal dispersion
- Distributed computing
- Excitation tailoring
- Less sensitive to errors than serial training

Disadvantages:
- Extra bookwork
- Must join all signals for model evaluations
Excitation Signals

For the aerodynamic identification process to capture a useful model, the dominant aerodynamics must be excited. The excitation is directly related to the input signal. The following criteria were identified as important to successful input signal design. The criteria are based on physics, performance and system model characteristics.

Criteria

The excitation must be consistent with the unsteady aerodynamic representation.

Static offset forces must be calculated

Excite the dominant unsteady aerodynamics while being kept in the "linear" aerodynamic range

Input and output dynamics must be excited

Excite the system within a useful frequency range
Advantages:
Easy to implement
Simple functional form
Broad PSD
Common flight-test signal

Disadvantages:
Inconsistent Boundary Conditions
Non intuitive excitation length
Holes in PSD
Chirp

Linear Frequency Sweep

\[ d(t) = \sin(\omega t^2) \]

\[ v(t) = 2\omega t \cos(\omega t^2) \]

Advantages:
Consistent with functional requirements
Flat PSD
Intuitive excitation length

Disadvantages:
Poor low frequency performance
Only captures analytic response
Can overexcite the system
DC Chirp

Linear Frequency Sweep with a DC Offset

\[ d(t) = -\frac{1}{2} \cos(\omega t^2) + \frac{1}{2} \]

\[ v(t) = \omega t \sin(\omega t^2) \]

Advantages:
Same advantages as the chirp
Improved low frequency excitation

Disadvantages:
Peak factor is twice the chirp's
Fresnel Chirp

The Fresnel chirp is the integral of the sine function. The Fresnel form is commonly seen in optics.

\[ d(t) = S(t) = \int \sin(\omega t^2) \]

\[ v(t) = \sin(\omega t^2) \]

Advantages:
Flat PSD for velocity
DC offset component

Disadvantages:
No closed form solution
Displacement PSD has holes
Offset Fresnel is impractical
Schroeder Sweep

The Schroeder form is based on a sum of cosine terms with a specified phasing. The form is analytic in time but not smooth in frequency.

\[
d(t) = \sum_{k=1}^{N} \sqrt{\frac{1}{2N}} \cos\left(\frac{2\pi kt}{T} - \frac{\pi k^2}{N}\right)
\]

\[
v(t) = \sum_{k=1}^{N} \frac{-2\pi k}{T} \sqrt{\frac{1}{2N}} \sin\left(\frac{2\pi kt}{T} - \frac{\pi k^2}{N}\right)
\]

Advantages:
Flat PSD
Optimal peak factor
Desirable low frequency response
Harmonic signal allows an arbitrary excitation length

Disadvantages:
Does not start from rest
Sensitivities to excitation method
An often proposed solution is to use a noise input signal.

**Advantages:**
- Flat PSD
- No limit on excitation length
- Excites entire flow dynamics

**Disadvantages:**
- Flat PSD only occurs in the limit
- Non intuitive excitation length
- Non deterministic causes BC problems
- Equal power means "sharp" changes
Artificial Noise

Contrary to intuition, adding noise to an existing signal is often desired for better high frequency modeling.
Motion Specification

Strict Specification

For strict motion specification, the motion boundary conditions at each timestep are determined from a numerical or analytical expression.

Advantages:
"Perfect" signals
Easy implementation

Disadvantages:
Not consistent in a discrete time sense

State Space Specification

For state space motion specification, the boundary conditions are updated simultaneously though the motion state vector. This exactly duplicates the actual discrete time CFD motion form.

Advantages:
Simultaneous BC updates
Input signals are filtered through a "structure" with mass.

Disadvantages:
Requires selecting a "structural mass"
Undesirable stopping conditions lead to motions that are nonlinear.
Model Evaluation

An evaluation methodology is needed to compare and select the "best" aerodynamic system model. Visual inspection is doomed; the evaluation must be based on a numerical measurement.

Evaluation based on Model Errors

\[ \chi^2 = \frac{||A \cdot \hat{x} - b||^2}{nstp} \]

Indicates the matrix solution quality

\[ RMS = \sqrt{\frac{1}{N} \sum_{k=0}^{N} (y(k) - \hat{y}(k))^2} \]

Indicates the model prediction quality
Model Evaluation

Evaluation based on Aeroelastic Stability

Using the model for coupled aerostructural predictions exercises the model's prediction characteristics in a realistic and intuitive manner. This evaluation is based on system eigenvalue predictions. The coupled aeroelastic plant matrix is:

\[
\begin{bmatrix}
G_s + q_\infty H_s D_a C_s & q_\infty H_s C_a \\
H_a C_s & G_a
\end{bmatrix}
\]
AGARD 445.6

Two Mode Structural Model

45 degree sweep
Aspect Ratio 2
60% taper ratio
NACA 65A004

First Bending Mode
9.6 Hertz

First Torsional Mode
38.2 Hertz
Stability Boundary

Mach Number vs. Dynamic Pressure [psi]
AGARD 445.6 Sensitivity Studies

Model Sensitivity Study
Eigenvalues in z-plane at Mach 0.499
Unit Circle, $|z|=1$

Mode 2
Increasing Density

Mode 1

Density Sweep

Stuctural Frequency Study

Structural Frequency Study

Stuctural Damping Study
Wing/Flap Control

Open Loop

Closed Loop

\[ k = \{3.5, 0, -0.01, -0.04\} \]

Closed Loop

\[ k = \{3.5, 0, -0.01, -0.05\} \]

Closed Loop Eigenvalues

\[ k = \{3.5, 0, -0.01, -0.05\} \]

Eigenvalues near the unit circle are causing a non-physical instability.
Conclusions

Improvements were made in the system identification routine.

A comparison of classical unsteady aerodynamic solutions was performed. The ARMA form remains the preferred system representation.

A parallel training method was developed. Parallel training allows for decoupled excitation and yields better system models.

An excitation signal survey was conducted. The DC-Chirp gave the most consistent results. High frequency limitations of the current signals were identified and evaluated.

Model quality "Evaluation criteria" were tested. The coupled stability prediction evaluation appears to give the best "model quality" indication.